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Elko and mass dimension one field of spin one half: causality and Fermi statistics

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We review how Elko arise as an extension of complex valued four-component Majorana spinors. This is followed by a discussion that constrains certain elements of phase freedom. A proof is reviewed that unambiguously establishes that Elko, and for that matter the indicated Majorana spinors, cannot satisfy Dirac equation. They, however do, as they must, satisfy spinorial Klein-Gordon equation. We then introduce a quantum field with Elko as its expansion coefficients and show that it is causal, satisfies Fermi statistics, and then refer to the existing literature to remind that its mass dimensionally is one. We conclude by providing an up-to-date bibliography on the subject.

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1. Elko

Elko^a are defined as the $(j, 0) \oplus (0, j)$ eigen-objects of the spin- j charge conjugation operator. They arose when one of us was at the Los Alamos National Laboratory in the early 1990s, and there was lot of excitement surrounding neutrino oscillations. Understanding their nature, and everything related to neutrinos was simply the inspiration of that exciting time. In order to understand neutrinos he had gotten Majorana's 1937 paper translated into English, and in that context he was studying Pierre Ramond's *primer* [1]. In the early chapters Ramond reminds that if $\phi(\mathbf{p})$ transforms as a massive $(0, 1/2)$ Weyl spinor then $\sigma_2 \phi^*(\mathbf{p})$ transforms as a massive $(1/2, 0)$ Weyl spinor. This observation allows for the introduction of a massive $(1/2, 0) \oplus (0, 1/2)$ four-component spinor

$$\lambda(\mathbf{p}) \stackrel{\text{def}}{=} \begin{pmatrix} \sigma_2 \phi^*(\mathbf{p}) \\ \phi(\mathbf{p}) \end{pmatrix}. \quad (1)$$

^aEigenspinoren des Ladungskonjugationsoperators, in German. For higher spins we will take the liberty of using the acronym Elko to mean 'eigen-objects' of the spin- j charge conjugation operator. This prevents the nomenclature to become too complicated.

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In Ramond's argument the 'magic of the Pauli matrices' plays a pivotal role. The interest of the first author at that time was to understand Majorana spinors and construct a higher spin counterpart of $\lambda(\mathbf{p})$ [nuclear physicists at LAMPF – the Los Alamos Meson Physics Facility – wanted to know how to describe higher spin resonances in a systematic manner]. The most natural extension of Ramond's observation where $\sigma_2\phi^*(\mathbf{p})$ is replaced by $J_y\phi_{(0,j)}^*(\mathbf{p})$ failed to reproduce the needed magic.^b The magic, the said author realized, instead lies in recognizing $\sigma_y = i\Theta$ with Θ interpreted as spin-1/2 Wigner time reversal operator. Then using the property that for any spin, the Wigner time reversal operator satisfies

$$\Theta_j \mathbf{J} \Theta_j^{-1} = -\mathbf{J}^* \quad (2)$$

allows the needed magic to happen, that is: if $\phi_{(0,j)}(\mathbf{p})$ is a left-transforming massive spin- j Weyl spinor belonging to the $(0, j)$ representation space, then $\zeta\Theta\phi_{(0,j)}^*(\mathbf{p})$, with $\zeta \in \mathbb{C}|\zeta^*\zeta = 1$, transforms as a right-handed massive spin- j Weyl spinor belonging to the $(j, 0)$ representation space. The demand that $(j, 0) \oplus (0, j)$ extension of (1)

$$\lambda_j(\mathbf{p}) \stackrel{\text{def}}{=} \begin{pmatrix} \zeta\Theta_j\phi_j^*(\mathbf{p}) \\ \phi_j(\mathbf{p}) \end{pmatrix} \quad (3)$$

be an eigen-object of the spin- j charge conjugation operator then determines the phase ζ .

For $j = 1/2$, one finds that $\zeta = \pm i$ and

$$\Theta = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (4)$$

For spin one half, one thus has not two, but four, Elko. The self-conjugate Elko are the standard Majorana spinors interpreted as complex-valued four-component spinors. While the existence of the other two Elko, no longer allows the set of four Elko to be interpreted as Weyl spinors in disguise. As far as spinors are concerned they are as fundamental as the well-known Dirac spinors: Elko are the eigenspinors of the charge conjugation operator while the Dirac spinors are the eigenspinors of the Parity operator.

While the 1937 Majorana field can be written in a form that resembles $\lambda_{j=1/2}(\mathbf{p})$ it is to be recalled that the field is still expanded in terms of the Dirac spinors, and not Elko.^c This superficial resemblance, if not properly clarified, can be a cause of much confusion. In fact the 1937 paper is a simple statement of the fact that if one sets $b_\sigma^\dagger(\mathbf{p}) = a_\sigma^\dagger(\mathbf{p})$ in the Dirac field then the charge carried by such a field is identically equal to zero. The 1937 field has nothing directly to do with Majorana

^b J_y is the $(2j+1) \times (2j+1)$ generator of rotation about the y -axis, and $\phi_{(0,j)}(\mathbf{p})$ is the left-transforming massive Weyl spinor of spin j .

^cWhen no confusion is likely to arise we will simply write $\lambda_{j=1/2}(\mathbf{p})$ as $\lambda(\mathbf{p})$.

spinors. To our knowledge, these arose only some two decades later in an attempt to reformulate Majorana field [2, 3].

2. Phases in Elko

For an arbitrary spin, there are $(2j+1)$ self-conjugate Elko, and an equal number of anti-selfconjugate Elko. Regarding $\phi_j(\mathbf{p})$ that appear in $\lambda_j(\mathbf{p})$, beyond the fact that they transform as left-handed spin- j Weyl objects, the Elko defining arguments place no other constraints on $\phi_j(\mathbf{p})$. If a choice of phases is made for the $\phi_j(\mathbf{p})$, then this introduces a relative phase between the right- and left-transforming components of $\lambda_j(\mathbf{p})$. A specific choice of these phases affects the physical and mathematical properties of spin- j Elko. While there is no a priori reason to take $\phi_j(\mathbf{p})$ as eigen-objects of $\mathbf{J} \cdot \hat{\mathbf{p}}$, this is the simplest choice that has been explored so far. One reason for this is that the same operator also enters in the rotation and boost transformations of the $(j, 0) \oplus (0, j)$ objects. The generators for the latter, \mathbf{K} , equal $\pm i$ times the generators for the former, \mathbf{J} .

While a general freedom of a global phase factor is not permitted for Elko, a restricted set of phases equal to ± 1 are allowed for each of the Elko without destroying their self/anti-selfconjugacy under the charge conjugation operator. This freedom, and the choice of the phases mentioned above, dramatically affects the locality anticommutators of the mass dimension one fermions of spin one half and their extensions to higher spins [4].

The 2005 publications mentioned above did not take these phases into account [5, 6]. The full appreciation of these phases emerged in a subsequent series of papers [7, 9]. They, in part, determine the locality structure of the originally-reported quantum field. The other part that enters the locality story is an observation that we will soon make below. But before we enter that thread let us show that the Dirac operator does not annihilate Elko.

3. Elko and the Dirac operator

The canonical spin one half Elko are obtained by choosing in equation (1), $\phi(k^\mu)$ as eigenspinors of the helicity operator $(1/2)\boldsymbol{\sigma} \cdot \hat{\mathbf{p}}$,

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{p}} \phi_{\pm}(k^\mu) = \pm \phi_{\pm}(k^\mu) \quad (5)$$

where

$$k^\mu = \begin{pmatrix} m \\ \frac{\mathbf{p}}{p} \Big|_{p \rightarrow 0} \end{pmatrix} \quad (6)$$

The phases associated with the Weyl spinors at ‘rest’ are then chosen as in Ref. [7, 8]. The arbitrary-momentum Elko now follow by boosting the $\lambda(k^\mu)$

$$\begin{aligned}\lambda(\mathbf{p}) &= \begin{pmatrix} \exp(\frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\varphi}) & \mathbb{O} \\ \mathbb{O} & \exp(-\frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\varphi}) \end{pmatrix} \lambda(k^\mu) \\ &= \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \mathbb{I} + \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m} & \mathbb{O} \\ \mathbb{O} & \mathbb{I} - \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m} \end{pmatrix} \lambda(k^\mu).\end{aligned}\quad (7)$$

This exercise yields a set of four Elko, two of them are self conjugate $\lambda_\pm^S(\mathbf{p})$, and the other two, $\lambda_\pm^A(\mathbf{p})$, are anti-selfconjugate under the action of the charge conjugation operator. The action of the Dirac operator on Elko can then be readily evaluated by using two observations, (a) the Elko at rest are

$$\lambda_\pm^S(k^\mu) = \begin{pmatrix} i\Theta\phi_\pm^*(k^\mu) \\ \phi_\pm(k^\mu) \end{pmatrix}, \quad \lambda_\pm^A(k^\mu) = \pm \begin{pmatrix} -i\Theta\phi_\mp^*(k^\mu) \\ \phi_\mp(k^\mu) \end{pmatrix} \quad (8)$$

with phases for $\phi_\pm(\mathbf{p})$ chosen as in Ref. [7, 8], and (b) that the helicity of $i\Theta\phi_\pm^*(k^\mu)$ is opposite to that of $\phi_\pm(k^\mu)$. Combined with equation (7) this yields

$$\lambda_\pm^S(p^\mu) = \sqrt{\frac{E+m}{2m}} \left(1 \mp \frac{p}{E+m}\right) \lambda_\pm^S(k^\mu) \quad (9)$$

$$\lambda_\pm^A(p^\mu) = \sqrt{\frac{E+m}{2m}} \left(1 \pm \frac{p}{E+m}\right) \lambda_\pm^A(k^\mu). \quad (10)$$

Now we can proceed further, and explicitly note that

$$\gamma_\mu p^\mu \lambda_+^S(p^\mu) = \left[E\gamma_0 + p \begin{pmatrix} \mathbb{O} & \boldsymbol{\sigma} \cdot \mathbf{p} \\ -\boldsymbol{\sigma} \cdot \mathbf{p} & \mathbb{O} \end{pmatrix} \right] \sqrt{\frac{E+m}{2m}} \left(1 - \frac{p}{E+m}\right) \lambda_+^S(k^\mu). \quad (11)$$

But since

$$\begin{pmatrix} \mathbb{O} & \boldsymbol{\sigma} \cdot \mathbf{p} \\ -\boldsymbol{\sigma} \cdot \mathbf{p} & \mathbb{O} \end{pmatrix} \lambda_+^S(k^\mu) = \gamma_0 \lambda_+^S(k^\mu) \quad (12)$$

we have

$$\gamma_\mu p^\mu \lambda_+^S(p^\mu) = (E+p) \sqrt{\frac{E+m}{2m}} \left(1 - \frac{p}{E+m}\right) \gamma_0 \lambda_+^S(k^\mu). \quad (13)$$

Next we use the identities

$$\gamma_0 \lambda_+^S(k^\mu) = i\lambda_-^S(k^\mu) \quad (14)$$

$$(E+p) \sqrt{\frac{E+m}{2m}} \left(1 - \frac{p}{E+m}\right) = \sqrt{\frac{E+m}{2m}} \left(1 + \frac{p}{E+m}\right) \times m \quad (15)$$

and

$$\sqrt{\frac{E+m}{2m}} \left(1 + \frac{p}{E+m}\right) \lambda_\pm^S(k^\mu) = \lambda_\pm^S(p^\mu) \quad (16)$$

to arrive at

$$\gamma_\mu p^\mu \lambda_+^S(p^\mu) = im \lambda_-^S(p^\mu). \quad (17)$$

Since similar results hold for the remaining three Elko, the above equation and the remaining three can be combined into the following four equations

$$\gamma_\mu p^\mu \lambda_\pm^{S,A}(p^\mu) = \pm im \lambda_\mp^{S,A}(p^\mu). \quad (18)$$

and arrive at the conclusion that Elko cannot satisfy the Dirac equation.^d

Equations (18) immediately combine to show that Elko do satisfy the spinorial Klein-Gordon equation

$$(\eta_{\mu\nu} p^\mu p^\nu \mathbb{I}_4 - m^2 \mathbb{I}_4) \lambda(p^\mu) = 0, \quad (19)$$

where $\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$.

4. From Elko to Mass dimension one field: causality and fermionic statistics

Since under the Dirac dual $\bar{\lambda}(p^\mu) \stackrel{\text{def}}{=} \lambda^\dagger(p^\mu) \gamma_0$, the scalar product $\bar{\lambda}(p^\mu) \lambda(p^\mu)$ identically vanishes, the route from here to the construction of mass dimension one fermionic fields requires the introduction of a new spinorial dual $\bar{\lambda}(p^\mu)$ in the $(1/2, 0) \oplus (0, 1/2)$ representation space. It can be found in reference [7, 8]. Once that is done one may calculate the Elko spin sums, and introduce a quantum field, $\mathbf{f}(x)$, with $\lambda_\pm^{S,A}(p^\mu)$ as its expansion coefficients

$$\mathbf{f}(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2mE(\mathbf{p})}} \sum_\alpha \left[a_\alpha(\mathbf{p}) \lambda_\alpha^S(\mathbf{p}) e^{-ip \cdot x} + b_\alpha^\dagger(\mathbf{p}) \lambda_\alpha^A(\mathbf{p}) e^{ip \cdot x} \right] \quad (20)$$

and its associated adjoint

$$\bar{\mathbf{f}}(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2mE(\mathbf{p})}} \sum_\alpha \left[a_\alpha^\dagger(\mathbf{p}) \bar{\lambda}_\alpha^S(\mathbf{p}) e^{ip \cdot x} + b_\alpha(\mathbf{p}) \bar{\lambda}_\alpha^A(\mathbf{p}) e^{-ip \cdot x} \right]. \quad (21)$$

In the definition of $\mathbf{f}(x)$ and $\bar{\mathbf{f}}(x)$ the statistics of the creation and destruction operators is left undetermined. However, the phases mentioned in section 2 must not be ignored.

To settle the statistics, consider two *space-like* separated events, x and y . For space-like separations temporal ordering of events is not necessarily preserved, and thus there exist two sets of inertial frame, ones in which $y^0 > x^0$ and the ones in which the reverse is true, $x^0 > y^0$. We call these sets of inertial frame as \mathcal{O} and \mathcal{O}' respectively. In \mathcal{O} , we calculate the amplitude for a particle to propagate from x to a space-like separated point y and in \mathcal{O}' the amplitude for an antiparticle to

^dThis result was first arrived at by Valeri Dvoeglazov, see references 10, 11. However, the derivation presented here is original and significantly more transparent.

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propagate from the point y to the space-like separated point x .^e Causality requires these two amplitudes may only differ by a phase:^f

$$\text{Amp}(x \rightarrow y, \text{particle})|_{\mathcal{O}} = e^{i\theta} \text{Amp}(y \rightarrow x, \text{antiparticle})|_{\mathcal{O}}, \quad (22)$$

with $\theta \in \mathbb{R}$. The calculated Elko spin sums force these two amplitudes to have opposite signs.^g Outline of this result is as follows

$$\begin{aligned} \text{Amp}(x \rightarrow y, \text{particle})|_{\mathcal{O}} &= \langle |\mathfrak{f}(y) \bar{\mathfrak{f}}(x)| \rangle \\ \text{Amp}(y \rightarrow x, \text{antiparticle})|_{\mathcal{O}'} &= \left[\langle |\bar{\mathfrak{f}}(x) \mathfrak{f}(y)| \rangle |_{\mathcal{O}} \right] \Big|_{(x-y) \rightarrow (y-x)} \\ &= -\text{Amp}(x \rightarrow y, \text{particle})|_{\mathcal{O}} \end{aligned} \quad (23)$$

Substituting $\text{Amp}(y \rightarrow x, \text{antiparticle})|_{\mathcal{O}'}$ from (23) into (22), gives $e^{i\theta} = -1$. The $\text{Amp}(y \rightarrow x, \text{antiparticle})|_{\mathcal{O}'}$ and $\text{Amp}(y \rightarrow x, \text{antiparticle})|_{\mathcal{O}}$ must yield the same probability, and thus they can differ by a phase alone:

$$\text{Amp}(y \rightarrow x, \text{antiparticle})|_{\mathcal{O}'} = e^{i\tilde{\theta}} \text{Amp}(y \rightarrow x, \text{antiparticle})|_{\mathcal{O}} \quad (24)$$

with $\tilde{\theta} \in \mathbb{R}$. However, one can go a step further and explicitly calculate the two indicated amplitudes in the above equation. We find them to be same. That is $e^{i\tilde{\theta}} = 1$. Using this result, (22) becomes:

$$\text{Amp}(x \rightarrow y, \text{particle})|_{\mathcal{O}} + \text{Amp}(y \rightarrow x, \text{antiparticle})|_{\mathcal{O}} = 0. \quad (25)$$

That is, $\mathfrak{f}(x)$ is fermionic

$$\langle |\{\mathfrak{f}(x), \bar{\mathfrak{f}}(y)\}| \rangle = 0, \quad (26)$$

and that the amplitude for a particle to propagate from x to y and an antiparticle to propagate from y to x in a given inertial frame are opposite.^h

To decipher the mass dimensionality associated with the set $\mathfrak{f}(x)$ and $\bar{\mathfrak{f}}(x)$ we calculate the Feynman-Dyson propagator, $S_{FD}(x - x') = i \langle |\mathfrak{T}[\mathfrak{f}(x') \bar{\mathfrak{f}}(x)]| \rangle$. Exploiting the just derived Fermi statistics, yields

$$\begin{aligned} S_{FD}(x - x') &= i \left[\text{Amp}(x \rightarrow x', \text{particle})|_{t' > t} - \text{Amp}(x' \rightarrow x, \text{antiparticle})|_{t > t'} \right] \\ &= i \left[\langle |\mathfrak{f}(x') \bar{\mathfrak{f}}(x)| \rangle \theta(t' - t) - \langle |\bar{\mathfrak{f}}(x) \mathfrak{f}(x')| \rangle \theta(t - t') \right]. \end{aligned} \quad (27)$$

^eFor doing the calculations we go from \mathcal{O} to \mathcal{O}' with a Lorentz transformation that interchanges $x \leftrightarrow y$: $(x - y) \rightarrow -(x - y)$.

^fThe essence of this argument can be traced back to Feynman [12] and Weinberg [13]. It differs from a later argument presented on the same subject by Weinberg in Ref. [14] where the pivotal role is played by the cluster decomposition principle.

^gIn order not to make our notation ‘heavy’ we are using the same symbols x and y , both in \mathcal{O} and \mathcal{O}' to mark the events under consideration. The distinction is then made unambiguous by explicitly referring the said frames.

^hIn all the above calculations we did not use the above arrived anti-commutativity.

In each of the two terms the indicated time orderings can be split into space-like and non space-like events. Because of the observation made following equation (26) the contributions for the former add. In consequence, $S_{FD}(x - x')$ allows propagation outside the light cone.

We have thus derived, rather than assumed, Fermi statistics for $\mathfrak{f}(x)$ and $\bar{\mathfrak{f}}(x)$. The $S_{FD}(x - x')$ has already been evaluated in the existing literature but its form now stands on firmer theoretical grounds. It endows field $\mathfrak{f}(x)$ with mass dimension one [5]–[7].ⁱ

5. Bibliography: Elko and Mass Dimension One Fermions

Elko and mass dimension one fermions are now an active area of research. A brief review of these investigations has been provided in the Foreword. Since the form of the latter does not allow a detailed bibliography we here list the relevant works [15] to [85].

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References

1. P. Ramond, *Field Theory: A Modern Primer*, Frontiers in Physics, Vol. 74, (Westview Press, 1990).
2. J. A. McLennan, “Parity nonconservation and the theory of the neutrino,” *Phys. Rev.* **106** (1957) 821.
3. K. M. Case, “Reformulation of the Majorana theory of the neutrino,” *Phys. Rev.* **107** (1957) 307.
4. C. Y. Lee, “Self-interacting mass-dimension one fields for any spin,” arXiv:1210.7916 [hep-th].
5. D. V. Ahluwalia and D. Grumiller, “Dark matter: A Spin one half fermion field with mass dimension one?,” *Phys. Rev. D* **72** (2005) 067701 [hep-th/0410192].
6. D. V. Ahluwalia and D. Grumiller, “Spin half fermions with mass dimension one: Theory, phenomenology, and dark matter,” *JCAP* **0507** (2005) 012 [hep-th/0412080].
7. D. V. Ahluwalia, C. Y. Lee, D. Schrittt and T. F. Watson, “Elko as self-interacting fermionic dark matter with axis of locality,” *Phys. Lett. B* **687** (2010) 248 [arXiv:0804.1854 [hep-th]].
8. D. V. Ahluwalia, C. Y. Lee and D. Schrittt, “Self-interacting Elko dark matter with an axis of locality,” *Phys. Rev. D* **83** (2011) 065017 [arXiv:0911.2947 [hep-ph]].
9. D. V. Ahluwalia and S. P. Horvath, “Very special relativity as relativity of dark matter: The Elko connection,” *JHEP* **1011** (2010) 078 [arXiv:1008.0436 [hep-ph]].

ⁱThe last two references incorporate the phases mentioned in section 2. These phases do not affect the mass dimensionality, but yield a dramatically improved locality structure.

10. V. V. Dvoeglazov, "Lagrangian for the Majorana-Ahluwalia construct," *Nuovo Cim. A* **108** (1995) 1467 [hep-th/9506083].
11. V. V. Dvoeglazov, "Neutral particles in light of the Majorana-Ahluwalia ideas," *Int. J. Theor. Phys.* **34** (1995) 2467 [hep-th/9504158].
12. R. P. Feynman, *Elementary Particles and the Laws of Physics* (Cambridge University Press, Cambridge, 1987).
13. S. Weinberg, *Gravitation and Cosmology: Principles and applications of the general theory of relativity* (John Wiley & Sons, New York, 1972).
14. S. Weinberg, *The Quantum theory of fields. Vol. 1: Foundations*, (Cambridge University Press, Cambridge, 1995).
15. R. da Rocha and W. A. Rodrigues, Jr., "Where are ELKO spinor fields in Lounesto spinor field classification?," *Mod. Phys. Lett. A* **21** (2006) 65 [math-ph/0506075].
16. C. G. Boehmer, "The Einstein-Cartan-Elko system," *Annalen Phys.* **16** (2007) 38 [gr-qc/0607088].
17. C. G. Boehmer, "The Einstein-Elko system: Can dark matter drive inflation?," *Annalen Phys.* **16** (2007) 325 [gr-qc/0701087].
18. C. G. Boehmer, "The Einstein-Elko system: Can dark matter drive inflation?," *Annalen Phys.* **16** (2007) 325 [gr-qc/0701087].
19. R. da Rocha and J. G. Pereira, "The quadratic spinor Lagrangian, axial torsion current, and generalizations," *Int. J. Mod. Phys. D* **16** (2007) 1653 [gr-qc/0703076 [GR-QC]].
20. C. G. Boehmer and D. F. Mota, "CMB anisotropies and inflation from non-standard spinors," *Phys. Lett. B* **663** (2008) 168 [arXiv:0710.2003 [astro-ph]].
21. R. da Rocha and J. M. Hoff da Silva, "From Dirac spinor fields to ELKO," *J. Math. Phys.* **48** (2007) 123517 [arXiv:0711.1103 [math-ph]].
22. D. V. Ahluwalia, C. Y. Lee, D. Schrittt and T. F. Watson, "Dark matter and dark gauge fields," arXiv:0712.4190 [hep-ph].
23. C. G. Boehmer, "Dark spinor inflation: Theory primer and dynamics," *Phys. Rev. D* **77** (2008) 123535 [arXiv:0804.0616 [astro-ph]].
24. D. Gredat and S. Shankaranarayanan, "Modified scalar and tensor spectra in spinor driven inflation," *JCAP* **1001** (2010) 008 [arXiv:0807.3336 [astro-ph]].
25. C. G. Boehmer and J. Burnett, "Dark spinors with torsion in cosmology," *Phys. Rev. D* **78** (2008) 104001 [arXiv:0809.0469 [gr-qc]].
26. R. da Rocha and J. M. Hoff da Silva, "ELKO, flagpole and flag-dipole spinor fields, and the instanton Hopf fibration," *Adv. Appl. Clifford Algebras* **20** (2010) 847 [arXiv:0811.2717 [math-ph]].
27. J. M. Hoff da Silva and R. da Rocha, "From Dirac action to ELKO action," *Int. J. Mod. Phys. A* **24** (2009) 3227 [arXiv:0903.2815 [math-ph]].
28. D. V. Ahluwalia, "Towards a relativity of dark-matter rods and clocks," *Int. J. Mod. Phys. D* **18** (2009) 2311 [arXiv:0904.0066 [gr-qc]].
29. S. Shankaranarayanan, "What-if inflaton is a spinor condensate?," *Int. J. Mod. Phys. D* **18** (2009) 2173 [arXiv:0905.2573 [astro-ph.CO]].
30. C. G. Boehmer and J. Burnett, "Dark energy with dark spinors," *Mod. Phys. Lett. A* **25** (2010) 101 [arXiv:0906.1351 [gr-qc]].
31. L. Fabbri, "Causal propagation for ELKO fields," *Mod. Phys. Lett. A* **25** (2010) 151 [Erratum-ibid. A **25** (2010) 1295] [arXiv:0911.2622 [gr-qc]].
32. L. Fabbri, "Causality for ELKOs," *Mod. Phys. Lett. A* **25** (2010) 2483 [arXiv:0911.5304 [gr-qc]].
33. C. G. Boehmer and J. Burnett, "Dark Spinors," arXiv:1001.1141 [gr-qc].
34. S. Shankaranarayanan, "Dark spinor driven inflation," arXiv:1002.1128 [astro-ph.CO].

35. H. Wei, “Spinor Dark Energy and Cosmological Coincidence Problem,” *Phys. Lett. B* **695** (2011) 307 [arXiv:1002.4230 [gr-qc]].
36. C. G. Boehmer, J. Burnett, D. F. Mota and D. J. Shaw, “Dark spinor models in gravitation and cosmology,” *JHEP* **1007** (2010) 053 [arXiv:1003.3858 [hep-th]].
37. G. Chee, “Stability of de Sitter solutions sourced by dark spinors,” arXiv:1007.0554 [gr-qc].
38. L. Fabbri, “Zero Energy of Plane-Waves for ELKOs,” *Gen. Rel. Grav.* **43** (2011) 1607 [arXiv:1008.0334 [gr-qc]].
39. K. E. Wunderle and R. Dick, “A Supersymmetric Lagrangian for Fermionic Fields with Mass Dimension One,” *Can. J. Phys.* **90** (2012) 1185 [arXiv:1010.0963 [hep-th]].
40. L. Fabbri, “The most general cosmological dynamics for ELKO matter fields,” *Phys. Lett. B* **704** (2011) 255 [arXiv:1011.1637 [gr-qc]].
41. L. Fabbri and S. Vignolo, “The most general ELKO Matter in torsional $f(R)$ -theories,” *Annalen Phys.* **524** (2012) 77 [arXiv:1012.4282 [gr-qc]].
42. M. Dias, F. de Campos and J. M. Hoff da Silva, “Exploring Elko typical signature,” *Phys. Lett. B* **706** (2012) 352 [arXiv:1012.4642 [hep-ph]].
43. A. Gillard and B. Martin, “Dark matter, Elko fields and Weinberg’s quantum field theory formalism,” *Rept. Math. Phys.* **69** (2012) 113 [arXiv:1012.5352 [hep-th]].
44. L. Fabbri, “Conformal Gravity with the most general ELKO Fields,” *Phys. Rev. D* **85** (2012) 047502 [arXiv:1101.2566 [gr-qc]].
45. R. da Rocha, A. E. Bernardini and J. M. Hoff da Silva, “Exotic Dark Spinor Fields,” *JHEP* **1104** (2011) 110 [arXiv:1103.4759 [hep-th]].
46. A. Basak and J. R. Bhatt, “Lorentz invariant dark-spinor and inflation,” *JCAP* **1106** (2011) 011 [arXiv:1104.4574 [astro-ph.CO]].
47. Y. X. Liu, X. N. Zhou, K. Yang and F. W. Chen, “Localization of 5D Elko Spinors on Minkowski Branes,” *Phys. Rev. D* **86** (2012) 064012 [arXiv:1107.2506 [hep-th]].
48. H. M. Sadjadi, “On coincidence problem and attractor solutions in ELKO dark energy model,” *Gen. Rel. Grav.* **44** (2012) 2329 [arXiv:1109.1961 [gr-qc]].
49. L. Fabbri and S. Vignolo, “A modified theory of gravity with torsion and its applications to cosmology and particle physics,” *Int. J. Theor. Phys.* **51** (2012) 3186 [arXiv:1201.5498 [gr-qc]].
50. A. E. Bernardini and R. da Rocha, “Dynamical dispersion relation for ELKO dark spinor fields,” *Phys. Lett. B* **717** (2012) 238 [arXiv:1203.1049 [hep-th]].
51. J. Lee, T. H. Lee, P. Oh, T. H. Lee and P. Oh, “Conformally-coupled dark spinor and FRW universe,” *Phys. Rev. D* **86** (2012) 107301 [arXiv:1206.2263 [gr-qc]].
52. T. H. Lee, “Some cosmological solutions of 5D Einstein equations with dark spinor condensate,” *Phys. Lett. B* **712** (2012) 6.
53. K. E. Wunderle and R. Dick, “Transformation properties and symmetry behaviour of ELKO spinors,” *Can. J. Phys.* **87** (2009) 909.
54. K. E. Wunderle and R. Dick, “Coupling of fermionic fields with mass dimensions one to the O’Raifeartaigh model,” arXiv:1208.4674 [hep-th].
55. R. da Rocha, J. M. Hoff da Silva and A. E. Bernardini, “Elko spinor fields as a tool for probing exotic topological spacetime features,” *Int. J. Mod. Phys. Conf. Ser.* **3** (2011) 133.
56. S. Kouwn, J. Lee, T. H. Lee and P. Oh, “Dark spinor model with torsion and cosmology,” arXiv:1211.2981 [gr-qc].
57. L. Fabbri, “Weakly-Interacting Massive Particles in Torsionally-Gravitating Dirac Theory,” *Int. J. Mod. Phys. D* **22** (2013) 1350071 [arXiv:1211.3837 [gr-qc]].
58. J. M. Hoff da Silva and R. da Rocha, “Unfolding Physics from the Algebraic Classification of Spinor Fields,” *Phys. Lett. B* **718** (2013) 1519 [arXiv:1212.2406 [hep-th]].

59. A. Basak, J. R. Bhatt, S. Shankaranarayanan and K. V. Prasantha Varma, "Attractor behaviour in ELKO cosmology," JCAP **1304** (2013) 025 [arXiv:1212.3445 [astro-ph.CO]].
60. H. M. Sadjadi, "Notes on teleparallel cosmology with nonminimally coupled scalar field," Phys. Rev. D **87** (2013) 064028 [arXiv:1302.1180 [gr-qc]].
61. R. da Rocha, L. Fabbri, J. M. Hoff da Silva, R. T. Cavalcanti and J. A. Silva-Neto, "Flag-Dipole Spinor Fields in ESK Gravities," J. Math. Phys. **54** (2013) 102505 [arXiv:1302.2262 [gr-qc]].
62. L. D. Sperana, "An Identification of the Dirac Operator with the Parity Operator," Int. J. Mod. Phys. D **23** (2014) 1444003 [arXiv:1304.4794 [math-ph]].
63. C. Y. Lee, "Symmetries of Elko and massive vector fields," arXiv:1306.5491 [hep-th].
64. E. Capelas de Oliveira, W. A. Rodrigues, Jr. and J. Vaz, Jr., "Elko spinor fields and massive magnetic like monopoles," Int. J. Theor. Phys. **53** (2014) 12, 4381 [arXiv:1306.4645 [math-ph]].
65. C. Y. Lee, "Symmetries of Elko and massive vector fields," arXiv:1306.5491 [hep-th].
66. J. A. Nieto, "Higher dimensional Elko theory," Rev. Mex. Fis. **60** (2014) 371 [arXiv:1307.1429 [gr-qc]].
67. S. Kouwn, J. Lee, T. H. Lee and P. Oh, "ELKO spinor model with torsion and cosmology," Mod. Phys. Lett. A **28** (2013) 1350121.
68. J. E. G. Silva and C. A. S. Almeida, "Kinematics and dynamics in a Bipartite-Finsler spacetime," Phys. Lett. B **731** (2014) 74 [arXiv:1312.7369 [hep-th]].
69. A. Alves, F. de Campos, M. Dias and J. M. Hoff da Silva, "Searching for Elko dark matter spinors at the CERN LHC," Int. J. Mod. Phys. A **30** (2015) 0006 [arXiv:1401.1127 [hep-ph]].
70. J. M. Hoff da Silva and S. H. Pereira, "Exact solutions to Elko spinors in spatially flat Friedmann-Robertson-Walker spacetimes," JCAP **1403** (2014) 009 [arXiv:1401.3252 [hep-th]].
71. R. T. Cavalcanti, J. M. Hoff da Silva and R. da Rocha, "VSR symmetries in the DKP algebra: the interplay between Dirac and Elko spinor fields," Eur. Phys. J. Plus **129** (2014) 11, 246 [arXiv:1401.7527 [hep-th]].
72. S. H. Pereira, A. P. S.S. and J. M. Hoff da Silva, "Some remarks on the attractor behaviour in ELKO cosmology," JCAP **1408** (2014) 020 [arXiv:1402.6723 [gr-qc]].
73. A. G. Nikitin, "Non-standard Dirac equations for non-standard spinors," arXiv:1403.0880 [math-ph].
74. C. Y. Lee, "The Lagrangian for mass dimension one fermionic dark matter," arXiv:1404.5307 [hep-th].
75. B. Agarwal, A. C. Nayak, R. K. Verma, and Pankaj Jain "ELKO as dark matter candidate," arXiv:1407.0797 [hep-ph].
76. A. P. S. S., S. H. Pereira and J. F. Jesus, "A new approach on the stability analysis in ELKO cosmology," Eur. Phys. J. C **75** (2015) 1, 36 [arXiv:1407.3401 [gr-qc]].
77. L. Fabbri and S. Vignolo, "ELKO and Dirac spinors seen from Torsion," Int. J. Mod. Phys. D **23** (2014) 1444001 [arXiv:1407.8237 [gr-qc]].
78. R. T. Cavalcanti, "Classification of singular spinor Fields and other mass dimension one fermions," Int. J. Mod. Phys. D **23** (2014) 1444002 [arXiv:1408.0720 [hep-th]].
79. R. da Rocha and J. M. Hoff da Silva, "Hawking radiation from Elko particles tunneling across black strings horizon," Europhys. Lett. **107** (2014) 50001 [arXiv:1408.2402 [hep-th]].
80. R. Rocha, "Exotic (Dark) Eigenspinors of the Charge Conjugation Operator," Springer Proc. Phys. **157** (2014) 439.
81. A. Alves, M. Dias and F. de Campos, "Perspectives for an Elko Phenomenology using

- monojets at the 14 TeV LHC,” arXiv:1410.3766 [hep-ph].
82. A. Basak and S. Shankaranarayanan, “Super-inflation and generation of first order vector perturbations in ELKO,” arXiv:1410.5768 [hep-ph].
83. R. Abłamowicz, I. Gonçalves and R. da Rocha, J. Math. Phys. **55** (2014) 10, 103501 [arXiv:1409.4550 [math-ph]].
84. L. Bonora, K. P. S. de Brito and R. da Rocha, “Spinor fields classification in arbitrary dimensions and new classes of spinor fields on 7-Manifolds,” arXiv:1411.1590 [hep-th].
85. I. C. Jardim, G. Alencar, R. R. Landim and R. N. C. Filho, “Solutions to the problem of ELKO spinor localization in brane models,” arXiv:1411.6962 [hep-th].